N 60123

Temp
192364

Paper to be presented at the Third Symposium on High-Speed Aerodynamics and Structures in San Diego on March 25-27, 1958

THEORETICAL EVALUATION OF THE PERFORMANCE OF AIR-BREATHING HYPERSONIC AIRPLANES

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COPY 1

There are two factors which make air-breathing vehicles potentially more efficient than rocket vehicles. One is the absence of the necessity for carrying an oxidant. The other is the possibility of converting an appreciable fraction of the fuel energy into kinetic energy of motion of the vehicle, rather than largely into kinetic energy of the exhaust. The latter effect decreases with increasing vehicle speed so that at a velocity of about half of satellite speed, depending upon the fuel, the air-breathing configuration becomes less efficient than the rocket even in principle. Practically, of course, the rocket is at present more efficient down to much lower speeds.

A number of ideas for improving the performance of air-breathing configurations have appeared in recent years, such as boundary layer control, supersonic combustion, and external heat addition. It is of interest to determine, as logically and as practically as we can, at the present time, the possible gains, limitations, and applications of some of these ideas. The object of this paper is to discuss the performance of air-breathing hypersonic aircraft in such a way that the following questions will be answered, or recalled to the mind of the audience for additional deliberation:

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(1) What are the fundamental quantities in terms of which performance can be measured and analyzed?

(2) What are the important design variables?

(3) What are some of the fundamental obstacles to development of air-breathing hypersonic aircraft?

(4) What greater performance than that now obtainable can we hope to achieve from fullest realization of potential gains?

Before treating these questions it should be established that in this paper consideration will be restricted to ordinary chemical fuels. It will be assumed that combustion occurs at a high enough density that the exhaust gas is nearly in chemical equilibrium. Under this condition there is no upper limit on the vehicle speed due to dissociation of the products of combustion or the air. Instead other factors which limit the speed such as friction drag and shock losses will be isolated for discussion.

Forces due to fuel mass flow will be neglected, in that the effect of fuel injection will be taken to be simple heat addition.

The aspects of performance to be discussed are defined in terms of flight paths which may be unnatural to rockets, but are appropriate for aircraft. The flight path is divided into three parts which are treated as independent missions. These are (1) acceleration to top speed, which from rocket terminology is called the burn-out velocity mission; (2) steady powered horizontal flight, as described by the original Bréguet range equation, which for brevity is called range; and (3) the unpowered glide. These three missions are independent if a separate stage is used for each. In any case it is convenient to analyze them as if they were independent. Comparisons will be made between the performance of air-breathing configurations and winged rockets for the burn-out velocity and range missions.

Since low sub-satellite speeds are of main interest for air-breathing configurations, the orbital centrifugal force will be suppressed.

Increases in weight necessitated by variable geometry will be tentatively assumed to be minor. The wing weight will be assumed to be a small enough fraction of the total weight that moderate variations in wing size do not affect the total weight.

In figure 1 we have expressions for the two aspects of performance we wish to evaluate, range and burn-out velocity. The Bréguet range equation given in figure 1 is the relation

$$\text{Range} = K\left(\frac{TV}{Q}\right)_{T=D}\frac{L}{D}\ \ln\left(\frac{W_i}{W_f}\right)\times 2000\ \text{miles}$$

The quantity (W_i/W_f) is the ratio of initial to final weight, and L/D is the usual ratio of wing lift to drag. The quantity $(TV/Q)_{T=D}$ is the engine over-all efficiency, which is the ratio of mechanical power developed by the engine to heat power supplied to the engine. The mechanical power developed is equal to the engine thrust (T) times vehicle velocity (V), and the heat power is denoted as (Q). In the range equation, the engine over-all efficiency must be evaluated under the condition of engine thrust equal to airplane drag. In that case, it is apparent that the two factors $(TV/Q)_{T=D}$ and L/D can be combined in the single ratio $(LV/Q)_{T=D}$. For configurations which develop lift by external combustion under the wing, or by engine exhaust deflection, we will see that the ratio $(LV/Q)_{T=D}$ cannot be factored into the two parts as it is here.

The factor (K) is a dimensionless fuel heat content parameter which expresses the fuel heat content per unit mass in terms of satellite kinetic energy per unit mass. The value of K is about 1.3 for gasoline, 1.8 for high energy boron fuels, and 4.0 for hydrogen.

The expression for the burn-out velocity given in figure 1 is the relation

$$\text{Burn-out velocity} = V_s \sqrt{K \left(\frac{\mathcal{T} V}{Q}\right)_{L=W} \ln \left(\frac{W_i}{W_f}\right)}$$

The rocket term, burn-out velocity, appears unnatural when applied to airplanes, but the meaning is clear. It is the maximum velocity the vehicle can attain before the fuel is exhausted in an accelerated flight starting from rest. Normally, of course, the maximum velocity of an airplane is determined by the operating limit of the thrust device rather than the fuel supply. However, if the final weight (W_f) is counted as the weight of the airplane at the end of acceleration, the relation given here applies to the accelerated part of the flight. Strictly, the relation given applies only if the over-all airplane efficiency $(\mathcal{T} V/Q)_{L=W}$, and the altitude are constant during the acceleration. However, only minor corrections are needed if altitude changes are counted as energy equivalent velocity changes. Here $\mathcal{T}$ is the net thrust (i.e., engine thrust minus airplane drag) under conditions of lift equal to airplane weight, K is the dimensionless fuel heat content parameter, and V_s is satellite velocity.

In figure 2 we see some slight changes in the expressions for range and burn-out velocity. The expressions for range and burn-out velocity listed in figure 2 are the relations

$$\text{Range} = K \left(\frac{\mathcal{L} V}{Q}\right)_{\mathcal{T}=0} \ln \frac{W_i}{W_f} \times 2000 \text{ miles}$$

and

$$\text{Burn-out velocity} = V_s \sqrt{K \left(\frac{\mathcal{T} V}{Q}\right)_{\mathcal{L}=W} \ln \left(\frac{W_i}{W_f}\right)}$$

When combustion under the wing or exhaust deflection are employed for lift augmentation, it is no longer possible to factor the lifting efficiency $(\mathcal{L}V/Q)_{\mathcal{T}=0}$ into the product of an engine efficiency and wing lift to drag ratio. Here $(\mathcal{L})$ is the total lift as distinguished from wing lift (L). Again $\mathcal{T}$ is the net thrust, which is zero in steady flight. The only change in the burn-out velocity expression is that the total lift $(\mathcal{L})$ rather than the aerodynamic lift alone supports the airplane weight.

It may be recalled that K is 1.3 for gasoline, and $(\mathcal{L}V/Q)_{\mathcal{T}=0}$ is about 2.0 for supersonic airplanes, so that the combination of these two factors is about 2.6. This number should be roughly independent of vehicle velocity in the high supersonic speed range assuming that the low speed range engine efficiency can be maintained.

In figure 3 we have expressions for the range and burn-out velocity of winged rockets. These expressions are the following:

$$\text{Range} = \frac{2IgV}{V_s^2}\,\frac{L}{D}\,\ln\left(\frac{W_i}{W_f}\right) \times 2000 \text{ miles}$$

and

$$\text{Burn-out velocity} = Ig\,\ln\left(\frac{W_i}{W_f}\right)$$

It should be recalled that, for present purposes, the range is defined as that part of the total range which is achieved during powered flight at constant velocity and altitude. The resulting expression is written so that the last two factors are the same as in the expression for airplanes. For a vehicle speed (V) equal to 14,000 feet per second and a specific impulse (I) equal to 300 seconds, the combination $2IgV/V_s^2 \times L/D$ is equal to 2.6 (which is equal to the corresponding number for supersonic airplanes burning gasoline). This factor for the rocket increases linearly with velocity showing that, at such speeds, rocket motors already developed are superior to gasoline ram jets which might be developed. However, this statement cannot be made so definite if hydrogen is the ramjet fuel, the corresponding number being about three times larger for hydrogen that it is for gasoline.

The expression for the burn-out velocity of the rocket, given here, applies when the acceleration is large compared to the acceleration due to gravity divided by (L/D). Comparison of the value of burn-out velocity with the corresponding value for ramjets will be delayed until after the burn-out velocity efficiency factor for ramjets, $(\mathcal{T}V/Q)_{\mathcal{L}=W}$, is further analyzed. In analogy with engine terminology, the quantity $(\mathcal{T}V/Q)_{\mathcal{L}=W}$ can also be called the over-all airplane efficiency.

The air-breathing hypersonic configuration selected for study is one with a ramjet engine and a wing. No external heat addition will be included, although this can be done in a way which defines the fundamental quantities involved.

In figure 4 we have a list of the design variables and expressions for the total forces and power as follows:

$\frac{\tilde{V}_e}{V}$ = ratio of exhaust velocity to vehicle velocity

θ = exhaust deflection from horizontal

$\frac{S}{A}$ = ratio of wing plan form area to engine inlet area

$$\mathcal{L} = \frac{1}{2}\rho V^2 A\left(2\frac{\tilde{V}_e}{V}\sin\theta + \frac{L}{D}C_D\frac{S}{A}\right)$$

$$\mathcal{T} = \frac{1}{2}\rho V^2 A\left(2\frac{\tilde{V}_e}{V}\cos\theta - 2 - \tilde{C}_{D,engine} - C_D\frac{S}{A}\right)$$

$$Q = \frac{1}{2}\rho V^2 AV\left\{\frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \tilde{\eta}_k\right]\right\}$$

These relations apply in the limit of large Mach numbers, although the same expressions apply for low Mach numbers when the definitions are changed slightly. The quantities for which the definitions change at low speed are marked with the cedilla. For example at low speed the quantity $\tilde{V}_e$ is equal to the exhaust velocity multiplied by the factor

$$\left(1 + \frac{2}{\gamma M_e^2}\right)$$

where M_e is the exhaust Mach number.

$\tilde{V}_e/V$ is the ratio of exhaust velocity to free stream or vehicle velocity. θ is the angle of deflection of the exhaust from horizontal. S/A is the ratio of wing plan form area to engine inlet area. These three variables have optimum values which depend on the design parameters and whether the mission is range or burn-out velocity.

The total lift ($\mathcal{L}$) is taken to be equal to the free stream dynamic pressure ($1/2\ \rho V^2$) times engine inlet area (A) times a quantity which is the sum of two terms. The first term is a contribution from deflection of the engine exhaust, and the second term is the wing lift expressed in terms of the wing L/D and drag coefficient (C_D). The wing lift coefficient and other wing design variables do not appear in this expression because it is assumed that L/D is already maximized with respect to such variables. It is perhaps obvious that the wing lift coefficient can be optimized independently of the other variables, although this is not true when external heat addition is considered.

The net thrust ($\mathcal{T}$) has three terms associated with the engine flow in addition to a term representing the wing drag. The engine drag coefficient based on engine inlet area ($\tilde{C}_{D,engine}$) includes the internal friction drag, and the wave drag due to bluntness at the inlet, but does not include shock losses which affect the internal flow uniformly. The latter losses are included by means of a kinetic energy parameter $\tilde{\eta}_k$ in the expression for the heat power supplied to the engine (Q). The kinetic energy efficiency for channel flow is defined by engine analysts as the ratio of the square of the velocity which would be obtained by isentropic expansion to inlet pressure of the flow at the exit divided by the square of the inlet velocity. A typical value of $\tilde{\eta}_k$ for supersonic diffusers is 0.9. This value has been obtained experimentally at Mach numbers up to about 5.0.

The quantity (η_t), appearing in the expression for heat power (Q), is the engine ideal thermodynamic cycle efficiency, which is a number between zero and one depending mainly upon the compression ratio. Values of η_t between 0.5 and 0.9 are typical. The expression for (Q), given here, was derived from the first two terms of an expansion in inverse powers of Mach number for the case of a supersonic constant area combustor. However, the expression applies to other cases, if the definitions of the quantities involved are altered slightly. An expression for heat power (Q) in terms of variables appearing in the expressions for the forces, such as the one given here, is the key to a simple design theory. The resulting theory can be used to extrapolate low speed knowledge to higher speeds. The relations for forces and power given in figure 4 can be regarded as a description of the lowest order effects. Refinements can be made by including additional terms.

Once the expressions for $\mathcal{L}$, $\mathcal{T}$, and Q are established, it is a straightforward procedure to form the particular efficiency ratio under consideration, $(\mathcal{L}V/Q)_{\mathcal{T}=0}$ for range or $(\mathcal{T}V/Q)_{\mathcal{L}=W}$ for burn-out velocity. The resulting expressions can then be maximized with respect to the design variables.

As an example we can look at the results for the burn-out velocity mission. Figure 5 is a plot of the optimum ratio of wing plan form area to engine inlet area as a function of the engine power coefficient (C_Q) and the design parameters. The engine power coefficient (C_Q) is defined as the heat power supplied to the engine made dimensionless through division by the free stream dynamic pressure, engine inlet area, and free stream velocity.

The combinations of design parameters listed in figure 5 are similarity parameters, by means of which the total original number of parameters is reduced to these three combinations. The combination

$$\frac{\sqrt{\tilde{\eta}_k}}{\sqrt{1 + \left(\frac{L}{D}\right)^2 \frac{L}{D} C_D}}$$

equal to 1.575 corresponds, for example, to kinetic energy efficiency ($\tilde{\eta}_k$) equal to 0.92, wing L/D equal to 6.0, and wing drag coefficient equal to 0.02. The value $\eta_t/\tilde{\eta}_k = 0.5$ corresponds, for example, to engine ideal thermodynamic cycle efficiency (η_t) equal to 0.48, and kinetic energy efficiency ($\tilde{\eta}_k$) equal to 0.92. The optimum wing size is plotted for several values of the third parameter $\left[\frac{C_W\sqrt{1 + (L/D)^2}}{\sqrt{\tilde{\eta}_k}}\right]$. C_W is the airplane weight coefficient based on free stream dynamic pressure and engine inlet area. If during the mission, the altitude is varied in order to maintain the optimum lift coefficient, this will cause the airplane weight coefficient to remain nearly constant also. As would be expected, increases of airplane weight result in increases of the optimum wing size. The optimum wing size relative to engine size is almost independent of engine power coefficient in contrast to the case when range rather than burn-out velocity is maximized. Since the attainable power coefficient decreases with increasing vehicle speed, it is gratuitous that the optimum wing size is not sensitive to this variation.

The dependence of figure 5 can be expressed analytically by the relation

$$\left(\frac{S}{A}\right)_{optimum} = \frac{\sqrt{\bar{\eta}_k}}{\sqrt{1 + \left(\frac{L}{D}\right)^2}\ \frac{L}{D}\ C_D}\left[\frac{C_W\sqrt{1 + \left(\frac{L}{D}\right)^2}}{\sqrt{\bar{\eta}_k}} - \sqrt{1 + \left(\frac{\eta_t}{\bar{\eta}_k}\ C_Q\right)}\right]$$

The optimum engine exhaust deflection was found to be given by the relation

$$\tan\ \theta_{opt} = \frac{1}{L/D}$$

In figure 6, the optimum value of the ratio of vehicle acceleration to the acceleration due to gravity for maximum burn-out velocity is plotted as a function of engine power coefficient and design parameters. It can be seen that as either airplane weight coefficient (C_W) or engine drag coefficient ($\tilde{C}_{D,engine}$) increase, the optimum acceleration decreases. If the engine power coefficient (C_Q) can be increased, the optimum acceleration naturally increases. For values of the parameters of interest at hypersonic speed, the optimum accelerations are all of the order of one half of the acceleration due to gravity or less in contrast to the large accelerations required for efficient operation of rockets.

The dependence of figure 6 can be expressed analytically by the relation

$$\left(\frac{dV/dt}{g}\right)_{optimum} = \frac{2\sqrt{\bar{\eta}_k}}{C_W}\ \frac{\sqrt{1 + \left(\frac{L}{D}\right)^2}}{L/D}\left\{\sqrt{1 + \frac{\eta_t}{\eta_k}\ C_Q} - \left[\frac{\left(1 + \frac{1}{2}\ \tilde{C}_{D,engine}\right)}{\sqrt{\bar{\eta}_k}}\ \frac{L/D}{\sqrt{1 + \left(\frac{L}{D}\right)^2}} + \frac{\frac{1}{2}\ C_W}{\sqrt{\bar{\eta}_k}\ \sqrt{1 + \left(\frac{L}{D}\right)^2}}\right]\right\}$$

Figure 7 is a plot of the over-all airplane efficiency for maximum burn-out velocity as a function of engine power coefficient and design parameters. Increases of airplane weight coefficient (C_W) or engine drag coefficient $\tilde{C}_{D,engine}$ lead to decreases in over-all efficiency. It can be seen in the figure that for given design parameters there is an optimum engine power coefficient. However, for all but the lowest values of the loss parameter

$$\left[\frac{\left(1+\frac{1}{2}\tilde{C}_{D,engine}\right)}{\sqrt{\tilde{\eta}_k}}\frac{L/D}{\sqrt{1+\left(\frac{L}{D}\right)^2}}+\frac{\frac{1}{2}C_W}{\sqrt{\tilde{\eta}_k}\sqrt{1+\left(\frac{L}{D}\right)^2}}\right]$$

the optimum occurs at unattainably high values of engine power coefficient. The over-all airplane efficiency $(\mathcal{G}V/Q)_{\mathcal{F}=W}$ is of the same nature as an engine over-all efficiency, and must lie between zero and one, as it does in the figure.

The dependence of figure 7 can be expressed analytically by the relation

$$\left(\frac{\mathcal{G}V}{Q}\right)_{\mathcal{F}=W}=2\,\frac{\eta_t}{\sqrt{\tilde{\eta}_k}}\,\frac{\sqrt{1+\left(\frac{L}{D}\right)^2}}{L/D}$$

$$\left\{\frac{\sqrt{1+\left(\frac{\eta_t}{\tilde{\eta}_k}C_Q\right)}-\left[\frac{\left(1+\frac{1}{2}C_{D,engine}\right)}{\sqrt{\tilde{\eta}_k}}\frac{L/D}{\sqrt{1+\left(\frac{L}{D}\right)^2}}+\frac{\frac{1}{2}C_W}{\sqrt{\tilde{\eta}_k}\sqrt{1+\left(\frac{L}{D}\right)^2}}\right]}{\left(\frac{\eta_t}{\tilde{\eta}_k}C_Q\right)}\right\}$$

It should be mentioned that there is a lower limit on L/D below which the relations given here do not apply. This limit corresponds to the value of L/D at which the optimum value of S/A is zero. From the form of the expression for the loss parameter, it might appear that the value of this parameter can be reduced by decreasing L/D. Actually, for values of L/D above the lower limit described above, decreasing L/D increases the loss parameter, as one would expect.

Figure 8 is a plot of the maximum attainable airplane velocity as a function of the loss parameter for two values of over-all airplane efficiency. For a ramjet engine, as is here under consideration, this velocity limit occurs because of the decrease with velocity of the attainable engine power coefficient. It may be recalled that the heat per unit mass of air which can be added to the air by a given fuel is essentially constant as long as nearly complete combustion can be maintained. In contrast, the kinetic energy of the air per unit mass of air passing through the engine increases as the square of the velocity. Consequently, the maximum attainable power coefficient decreases with vehicle velocity according to the relation

$$\left.\begin{aligned} C_{Q,\mathrm{maximum}} &= \frac{1}{9}\left(\frac{V_s}{V}\right)^2 \text{ hydrogen} \\ C_{Q,\mathrm{maximum}} &= \frac{1}{13}\left(\frac{V_s}{V}\right)^2 \text{ gasoline} \end{aligned}\right\}$$

Taking this factor into account, an over-all airplane efficiency of 0.333 can be maintained only to a certain maximum velocity, which depends on the loss parameter as indicated in the figure. Also shown is the maximum velocity to which over-all efficiencies of 0.111 and 0 can be maintained. The maximum velocity for zero efficiency is the actual maximum velocity of the configuration, since no further acceleration is possible, when the over-all efficiency becomes zero.

Although the simple design theory used in this paper was derived for ramjet engines at hypersonic speeds, it also applies to turbojet engines and low speeds. In the remainder of the paper the predictions of the theory in the whole speed range will be discussed.

Figure 8 can be used to illustrate several of the fundamental obstacles to development of air-breathing hypersonic flight. A typical value for the weight coefficient based on engine inlet area of a supersonic interceptor is six. The weight coefficient for a turbojet engine alone is typically one. If the engine alone has this large a value, the value for the airplane could hardly be held to less than three. With an L/D of six, $\tilde{\eta}_k$ equal to one, and $\tilde{C}_{D,engine}$ equal to zero, the loss parameter due to weight alone, would then be about 1.25. Figure 8 indicates that an over-all airplane efficiency of 0.333 could not be achieved at any speed for such a value of the loss parameter. The maximum velocity for an over-all efficiency of 0.111 would be about 7000 feet per second assuming that the low speed values of cycle and combustion efficiency could be maintained and other losses remain small.

Ramjet engines can be made lighter than turbojet engines, such that the weight coefficient may not be the determining factor for maximum velocity of aircraft with such engines. However, assuming an engine drag coefficient of 0.1, a kinetic energy efficiency of 0.92, and zero weight loss, the value of the loss parameter is about 1.09. Then for an over-all airplane efficiency of 0.111, the velocity is limited by friction drag and shock losses to less than about 12,000 feet per second.

Although these considerations are binding for the immediate future, they may not remain so. For example, external heat addition offers interesting possibilities of reducing the weight loss. Reduction of friction drag inside the engine would also lead to the possibility of reducing internal shock losses by increasing the length per unit inlet dimension. In fact by studying the factors which affect the value of the loss parameter, it can be seen that there is a minimum attainable value which is determined essentially by air friction in the engine and in the wing flow. Any significant decrease in skin friction effects below those which are ordinarily experienced, would profoundly affect the performance possibilities of air-breathing configurations.

In figure 9 the other extreme of performance possibilities which cannot be exceeded by innovation are depicted. Burn-out velocity is plotted as a function of the required ratio of initial to final weight for hydrogen ramjets, gasoline ramjets and rockets. The over-all airplane efficiencies are taken to be 1/3 and a specific impulse of 300 is assigned to the rocket. The curve labeled gasoline ramjet also applies to a hydrogen ramjet with an over-all airplane efficiency of 0.111. Since the value of over-all airplane efficiency is assumed, the orbital centrifugal force is not neglected in figure 9.

At first sight the ramjets look very good in this type of plot, since it is indicated that with an over-all airplane efficiency of 1/3, a hydrogen ramjet can fly to satellite velocity for a mass ratio of about the same order as airplanes occasionally employ. Such performance is possible as far as the first and second laws of thermodynamics are concerned. However it may be recalled from the previous figure that the required loss parameter at the higher velocities is much lower than airplanes normally achieve. Also at the higher speeds other factors which have not been considered here are decisive, such as cooling requirements, and the requirements for supersonic combustion.

It is of interest to compare rockets with airplanes which are at present feasible as first stages for launching satellites or other high-speed vehicles. The proposed Vanguard flight shown in figure 9 is a typical flight for present rockets. After burn-out of the first stage, motors and tanks are ejected, which is represented by a displacement to the right. The second stage fires and achieves a higher velocity, which is followed by a displacement representing further ejection of motors and tanks in preparation for firing the third stage.

Consider an airplane which can fly to a velocity of 4000 feet per second. Such a velocity can be reached for a small mass ratio of the order of that indicated in figure 9 for ramjets. However even if the airplane is specifically designed for this purpose, perhaps half of it is useless for higher speeds and should be ejected. The resulting displacement terminates at point approximately on the Vanguard flight, which means that the starting weight of the airplane is no less than that of the corresponding rocket. For a single flight, cost factors, not considered here, make the development of such specialized airplanes uneconomical. However if the airplane can be landed and used to launch several missiles successively, all of the airplane cost should not be charged to a single flight. Taking this factor into account, it may be possible to approach the savings indicated in figure 9 for ramjets.

In conclusion, it can be said that simple mass and energy considerations indicate the air-breathing vehicle to be potentially efficient for launching missiles or satellites at regular time intervals.

Efficient operation at hypersonic speed probably requires airplane staging because of the large weight of the turbojet engines needed at low speed.

A practical method for reducing skin friction would greatly enhance the performance possibilities of air-breathing vehicles.

TABLE OF SYMBOLS

A	engine inlet area
C_D	wing drag coefficient (equal to $D/\frac{1}{2}\rho V^2 S$)
$\tilde{C}_{D,engine}$	engine drag coefficient (equal to $D_{engine}/\frac{1}{2}\rho V^2 A$)
C_Q	engine power coefficient (equal to $Q/\frac{1}{2}\rho V^2 AV$)
C_W	airplane weight coefficient (equal to $W/\frac{1}{2}\rho V^2 A$)
D	wing drag
D_{engine}	engine drag
g	acceleration due to gravity (equal to 32.2 ft per sec^2)
I	specific impulse of rocket (lbs thrust per lb of fuel per sec)
K	heat content parameter (equal to 1.3 for gasoline)
L	wing lift
$\mathcal{L}$	total lift
Q	heat power supplied to the engine
S	wing plan form area
T	engine thrust
$\mathcal{T}$	net thrust (equal to engine thrust minus wing drag)
V	free stream or vehicle velocity
$\tilde{V}_e$	exhaust velocity
V_s	satellite velocity (equal to 26,100 ft/sec)
W	instantaneous airplane weight
W_i	initial airplane weight
W_f	final airplane weight
θ	engine exhaust deflection from horizontal
$\tilde{\eta}_k$	engine kinetic energy efficiency
η_t	engine thermodynamic ideal cycle efficiency
ρ	free stream density

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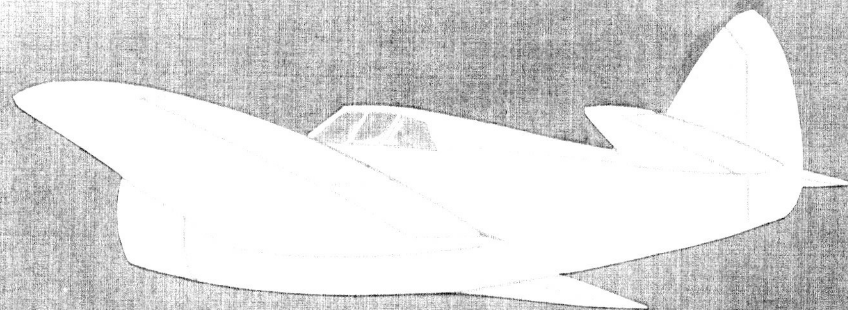
RANGE and BURNOUT VELOCITY RELATIONS FOR AIRPLANES IN HORIZONTAL MOTION -I



$$\text{Range} = K\left(\frac{TV}{Q}\right)_{T=D} \times \frac{L}{D} \times \ln\frac{W_i}{W_f} \times 2000 \text{ Miles}$$

$$\text{Burnout Velocity} = V_S\sqrt{K\left(\frac{TV}{Q}\right)_{L=W} \ln\left(\frac{W_i}{W_f}\right)}$$

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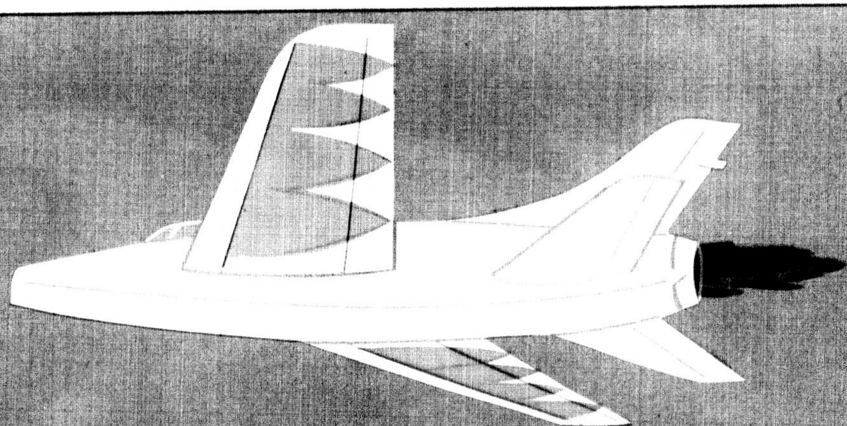
RANGE and BURNOUT VELOCITY RELATIONS FOR AIRPLANES IN HORIZONTAL MOTION-II



$$\text{Range} = K\left(\frac{LV}{Q}\right)_{T=0} \times \ln \frac{W_i}{W_f} \times 2000 \text{ Miles}$$

$$\text{Burnout Velocity} = V_s \sqrt{K\left(\frac{TV}{Q}\right)_{L=W} \ln \frac{W_i}{W_f}}$$

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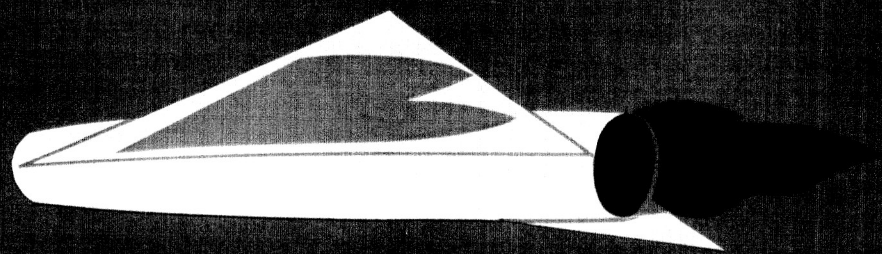
RANGE and BURNOUT VELOCITY EXPRESSIONS FOR RAMJETS IN HORIZONTAL MOTION



$$\text{Range} = K\left(\frac{\mathcal{L}V}{Q}\right)_{\mathcal{T}=0} \times \ln\left(\frac{W_i}{W_f}\right) \times 2000 \text{ Miles}$$

$$\text{Burnout Velocity} = V_s \sqrt{K\left(\frac{\mathcal{T}V}{Q}\right)_{\mathcal{L}=W} \ln\left(\frac{W_i}{W_f}\right)}$$

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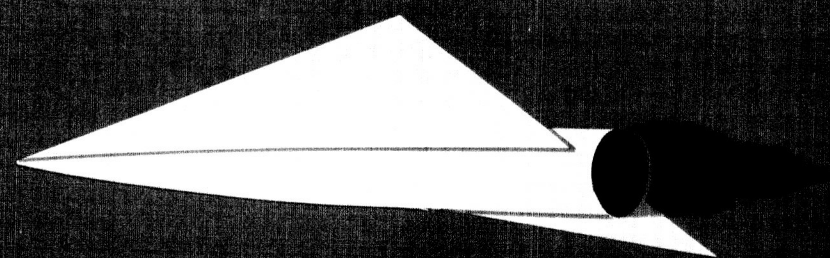
RANGE and BURNOUT VELOCITY EXPRESSIONS FOR ROCKETS IN HORIZONTAL MOTION



$$\text{Range} = \frac{2IgV}{V_s^2} \times \frac{L}{D} \times \ln \frac{W_i}{W_f} \times 2000 \text{ Miles}$$

$$\text{Burnout Velocity} = Ig \ln \frac{W_i}{W_f}$$

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TOTAL FORCES AND POWER

$\frac{\tilde{V}_e}{V}$ = RATIO OF EXHAUST VELOCITY TO FREE STREAM VELOCITY

θ = ENGINE EXHAUST DEFLECTION FROM HORIZONTAL

$\frac{S}{A}$ = RATIO OF WING PLANFORM AREA TO ENGINE INLET AREA

$$\mathcal{L} = \frac{1}{2}\rho V^2 A\left\{2\frac{\tilde{V}_e}{V}\sin\theta + C_L\frac{S}{A}\right\}$$

$$\mathcal{T} = \frac{1}{2}\rho V^2 A\left\{2\frac{\tilde{V}_e}{V}\cos\theta - 2 - \tilde{C}_{D_{engine}} - C_D\frac{S}{A}\right\}$$

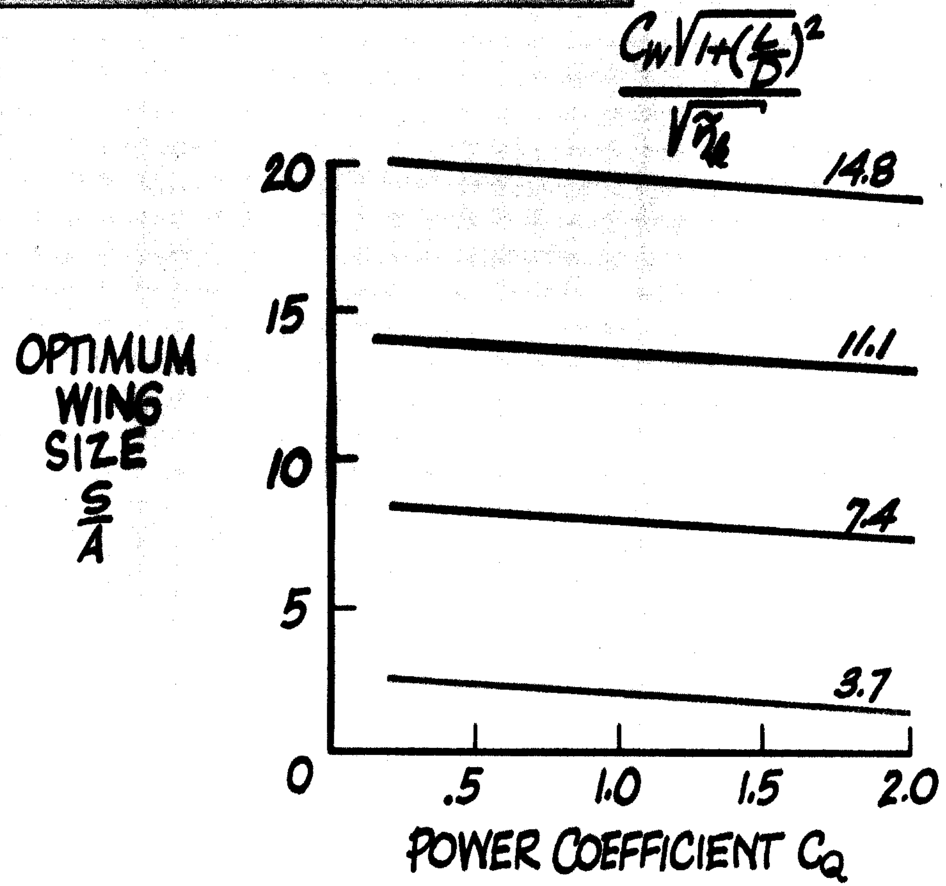
$$Q = \frac{1}{2}\rho V^2 A V\left\{\frac{1}{\eta_t}\left[\left(\frac{\tilde{V}_e}{V}\right)^2 - \eta_k\right]\right\}$$

RANGE EFFICIENCY RATIO = $\left(\frac{\mathcal{L}V}{Q}\right)_{\mathcal{T}=0}$

BURNOUT VELOCITY EFFICIENCY RATIO = $\left(\frac{\mathcal{T}V}{Q}\right)_{\mathcal{L}=W}$

A-23673-6

WING SIZE FOR MAXIMUM BURNOUT VELOCITY



$$\frac{\sqrt{\eta_k}}{\sqrt{1+(\frac{L}{D})^2}\,\frac{L}{D}C_D} = 1.575$$

$$\frac{\eta_t}{\eta_k} = .500$$

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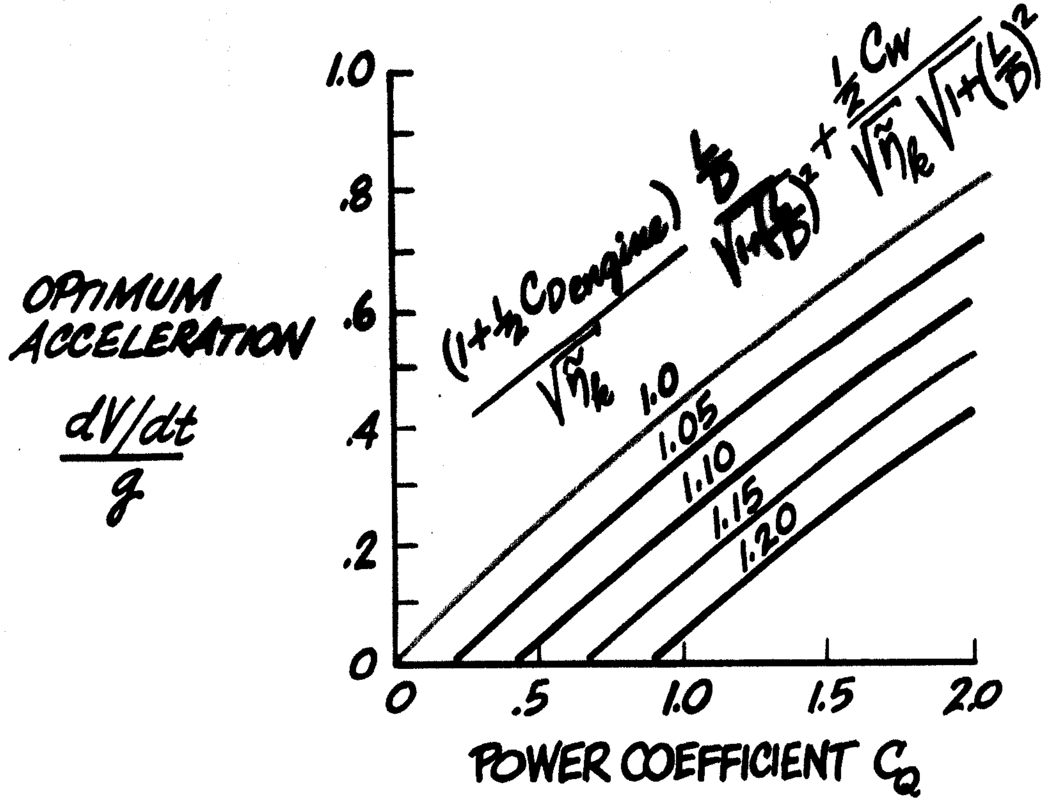
RAMJET ACCELERATION FOR MAXIMUM BURNOUT VELOCITY



$$\frac{\sqrt{\tilde{\eta}_k}}{C_W}\sqrt{1+\left(\frac{L}{D}\right)^2} = .972$$

$$\frac{\eta_t}{\tilde{\eta}_k} = .500$$

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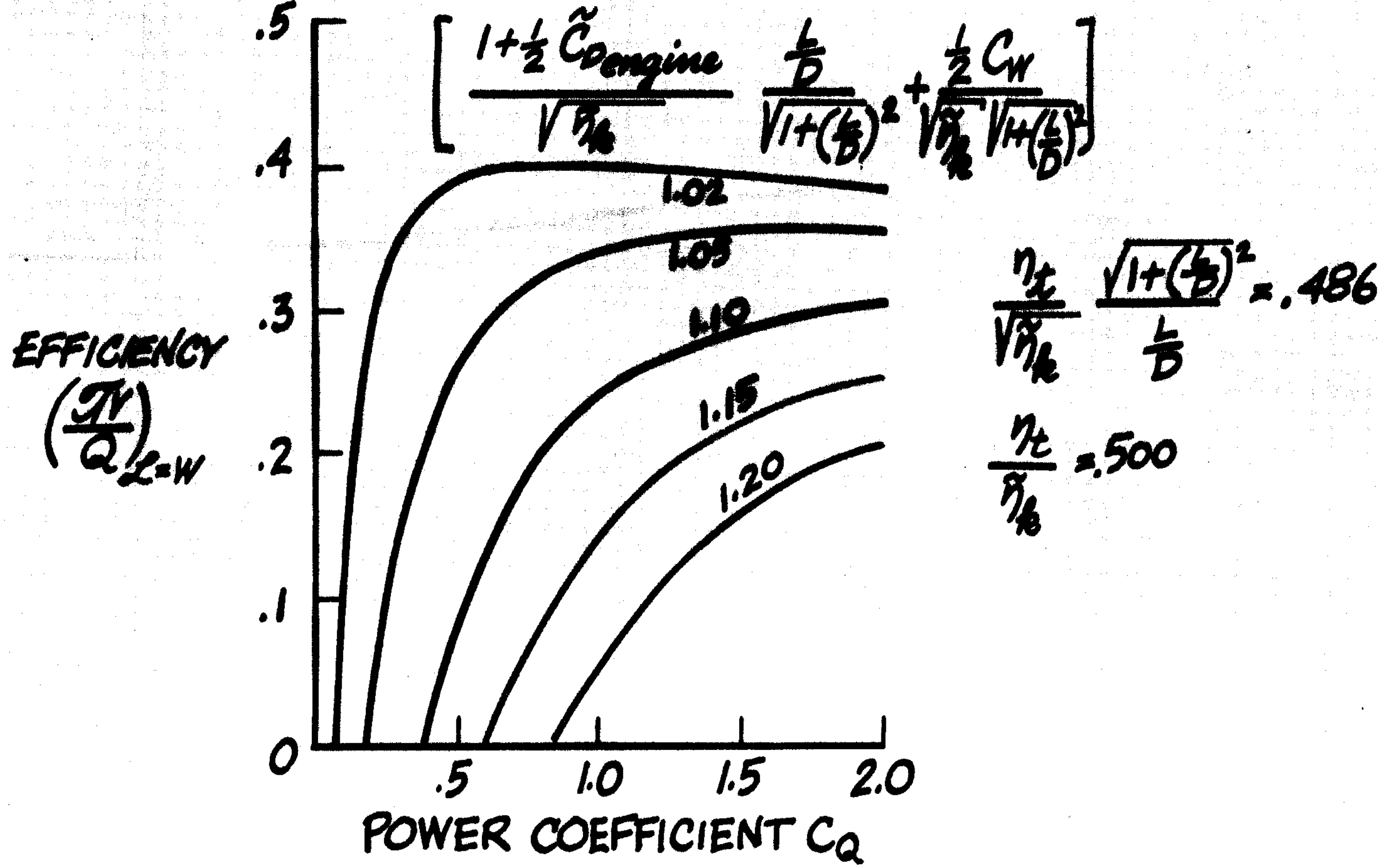
OVERALL EFFICIENCY FOR PRODUCING BURNOUT VELOCITY



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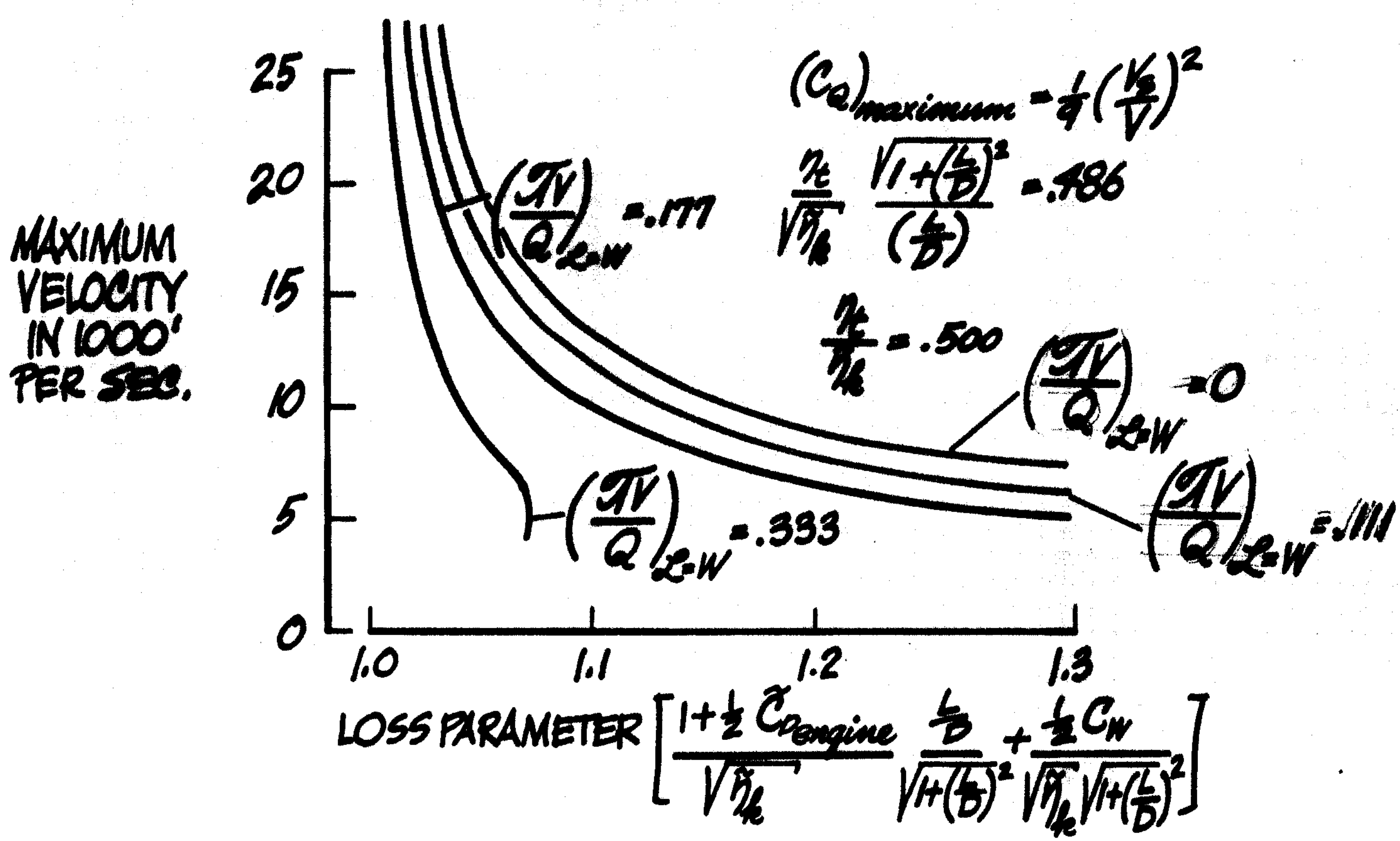
MAXIMUM VELOCITY OF HYDROGEN RAMJET VEHICLE



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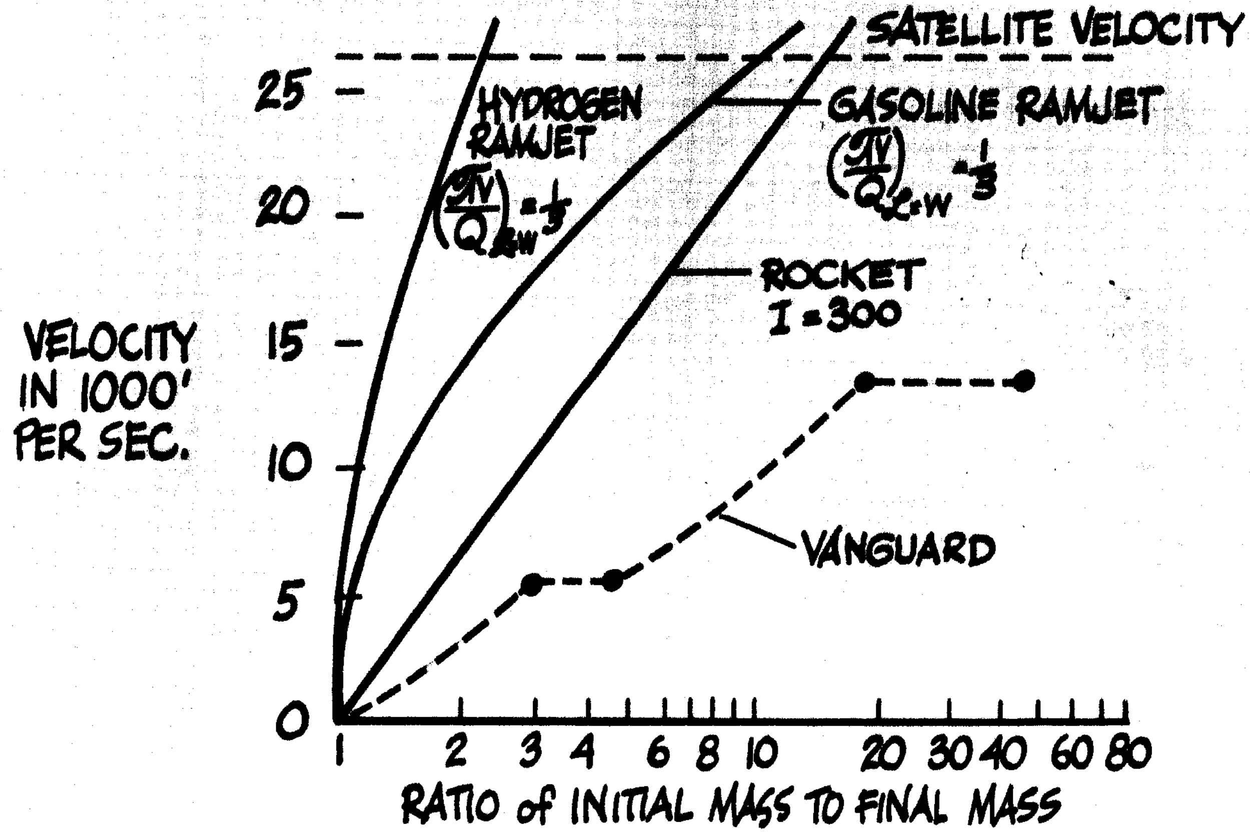
BURNOUT VELOCITY



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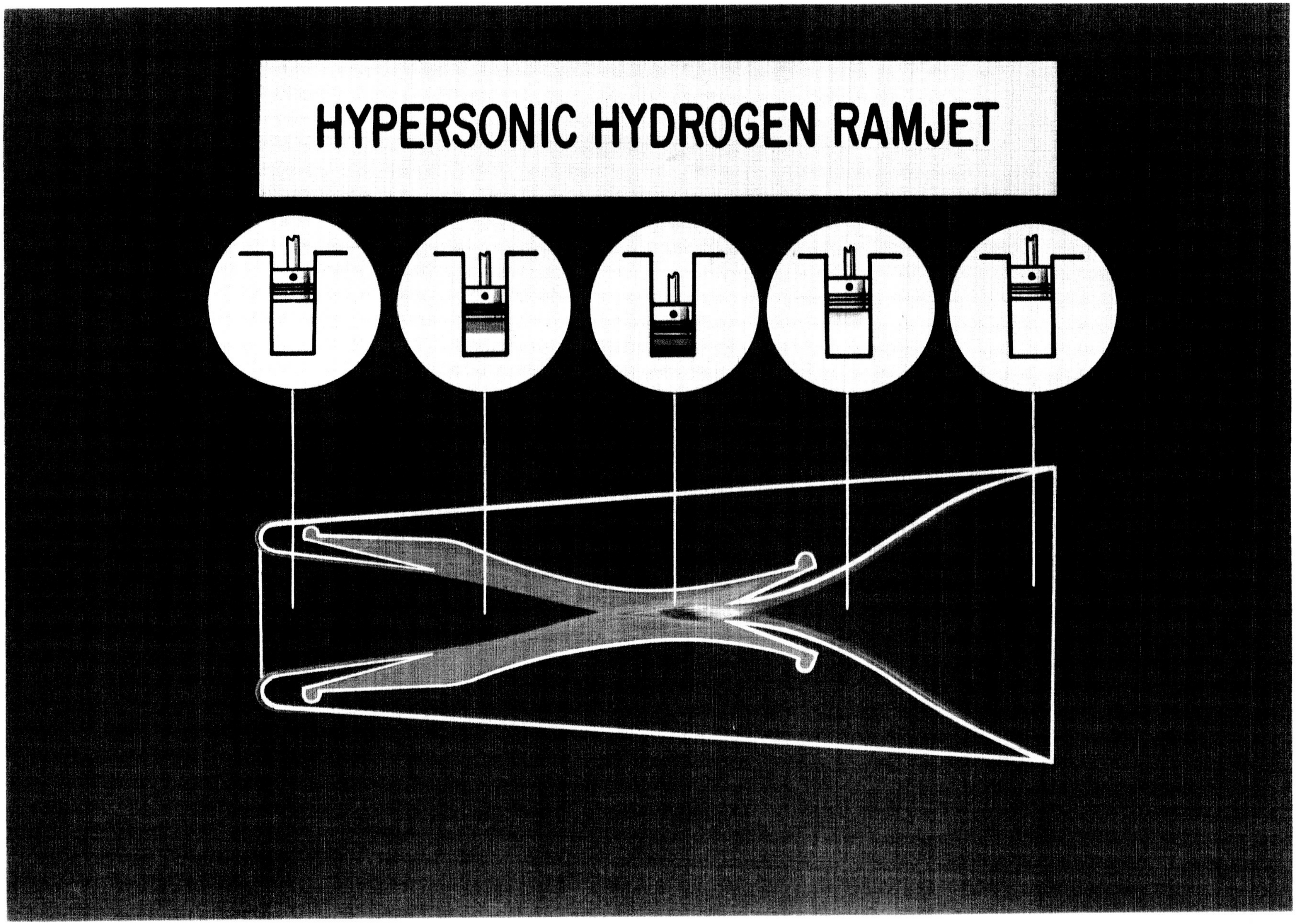

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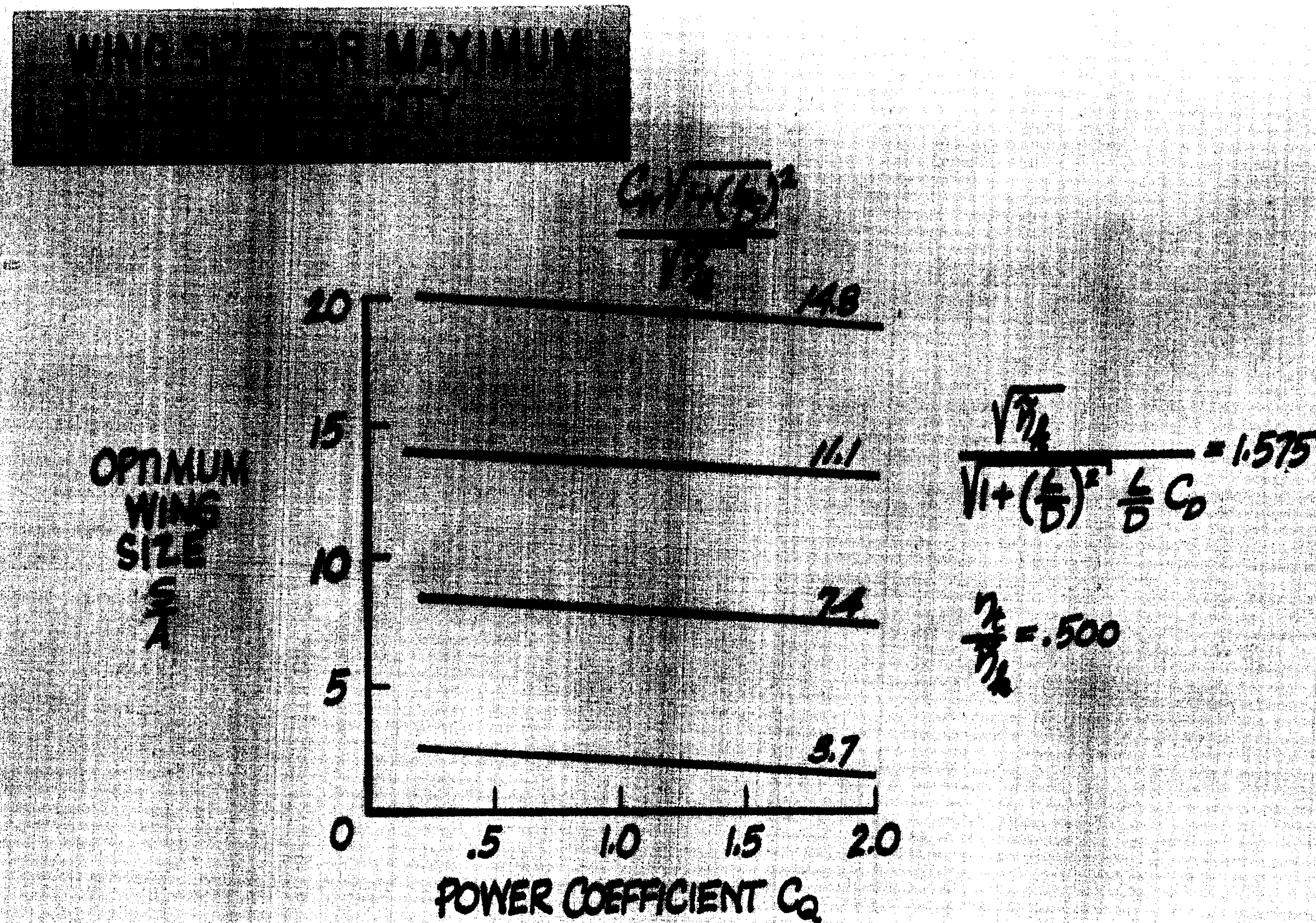

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